

## Particle mobility calculator:

an Excel workbook for easy conversions  
between aerosol particle size and mobility

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The conversions in the workbook assume the aerosol particles to be spherically symmetric, singly charged and located in the air. The symbols and units of physical quantities are explained in the control form (see the screenshot below). The problematic concept of nanoparticle diameter is not discussed, the user can find explanations from publications e.g. Mason (1984), Tammet (1995) and Tammet (2012b).

Depending on the Excel settings, opening the file "Particle mobility calculator.xls" may display a question about admissibility of macros. If you are sure that the file is authentic, enable macros. Now the spreadsheet should appear and you can press Ctrl+M (a less convenient universal alternative is Alt+F8 and Run) to launch the control form:

**Aerosol particle size-mobility conversions**

**Brief instruction**  
**NB: Extended instruction is available as a document "Particle mobility calculator.pdf"**  
The user can allocate values of different quantities in columns A...Z of the worksheet free from restrictions. A quantity (air temperature, particle diameter etc) may have values:  
\* as a constant not shown in the worksheet (e.g.  $p = 1013$ ),  
\* prepared by user in a column of the worksheet,  
\* generated by means of this macro as an arithmetic or geometric progression.  
When the values of arguments are defined, the required quantity (mobility, diameter or mass) can be calculated for all rows of the worksheet. Fill in the necessary control data, click button "Read control data" and a button of operation e.g.  $d \Rightarrow Z$ . Then the control data can be changed, reread, and a

**Number of rows**  
First row of the worksheet contains column headers. Data rows are numbered 2...n+1. Value of n is taken from the box:  
16

**Symbols and units**  
p = air pressure : mb  
T = air temperature : C  
rho = particle density : g cm<sup>-3</sup>  
d = particle diameter : nm  
m = particle mass : amu  
Z = particle mobility : cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup>

**Generate data**  
Column name: A header: d:nm first value: 0.3162277 last value: 10  
Generate arithmetic progression Generate geometric progression

**Slip factor coefficients**  
A = 1.165 B = 0.483 C = 0.997  
Pre-filled according to ISO 15900, can be changed by user

**Extension of particle diameter (nm) or code**  
0.37 Particle diameter in Millikan equation is replaced by sum of geometric diameter and extension. Ku and de la Mora (2009) recommended constant extension 0.3. Diameter of a N<sub>2</sub> molecule is 0.37. A value > 3 is considered as a code of variable extension. Code 1995 calls the old algorithm by Tammet (1995), code 2012 the simplified approximation by Tammet (2012), and code 9999 calls an user-written VBA function User\_extension (p, T, rho, d).

**Quantities**

| Symbol | Value | Header |
|--------|-------|--------|
| p      | 1000  | p      |
| T      | 10    | T      |
| rho    | 2     | rho    |
| d      | A     | d      |
| m      | B     | m      |
| Z      | C     | Z      |

Constant value must be presented with a number and tabulated value with the name of the column A...Z

Read control data  $d \Rightarrow m$   $m \Rightarrow d$   $d \Rightarrow Z$   $Z \Rightarrow d$  EXIT

While the control form is open, the spreadsheet will stay frozen. If you wish to resize or move the Excel window or enter data into the spreadsheet, click "EXIT" in the control form, perform the necessary actions and press Ctrl+M again to reopen the control form.

The control form contains 21 control boxes and 8 command buttons. The control boxes are pre-filled with sample data, which can be edited and replaced by the user with necessary data.

The frame "Number of rows" contains one control box, which affects any action launched with the command buttons. This box should never be left empty.

The frame "Generate data" helps to fill columns with progressions. It contains 4 control boxes and two command buttons. The progressions can be increasing, constant, or decreasing. A constraint is that the first and last values of a geometric progression must be positive numbers. If you are not satisfied with the first result, then the operation can be repeated several times with changed control data. If you do not change the sample data and click "Generate geometric progression", then the cell A1 will contain the header "d:nm" and the cells A2...A17 will contain a logarithmic series of particle diameters: 0.316, 0.398, 0.501....., 7.94, and 10. This is a standard way to introduce typical problems about size-mobility functions.

The lower part of the control form below the frame "Generate data" contains 16 control boxes and 6 command buttons, which allow carry out 4 possible conversions:

| Button  | Arguments    | Result | Constraints  |
|---------|--------------|--------|--|
| d ==> m | d, rho       | m      | no   |
| m ==> d | m, rho       | d      | no   |
| d ==> Z | d, p, T, rho | Z      | $0.3 \text{ nm} \leq d \leq 1000 \text{ nm}$   |
| Z ==> d | Z, p, T, rho | d      | $0.0001 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1} \leq Z \leq 5 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$ |

The four conversion buttons are initially deactivated and shown in gray. They will activate with a click on the button "Read control data". This button checks the contents of all control boxes and displays the values on the status bar of the spreadsheet. After any conversion the conversion buttons will again be deactivated and the user must click "Read control data" before every new conversion operation.

Diameter-mobility conversions can be carried out with several options:

- plain Millikan equation (diameter extension = 0),
- constant-updated Millikan equation with arbitrary diameter extension up to 3 nm,
- function-updated Millikan equation according to Tammet (2012a),
- function-updated Millikan equation with user-written algorithm of diameter extension,
- approximation by Tammet (1995).

Three slip factor coefficients are pre-filled according to the standard ISO 15000. The values written in the control boxes are used in all mobility models including the model with the code 1995. This means a small deviation from the original algorithm published in paper by Tammet (1995), where different slip factor coefficients were used.

The frame "Quantities" contains 12 control boxes for six quantities p, T, rho, d, m, and Z. Any quantity can be specified as a constant or as a tabulated variable, which values are written in a certain column of the spreadsheet. If the value box contains a number then the quantity is interpreted as a constant. If the value box contains a letter A...Z (or a...z, the letters are not case sensitive) then the quantity is interpreted as a variable and its values are addressed to the corresponding column of the spreadsheet. The column of the result of conversion may be initially empty of filled. In the last case the old content of the column will be replaced with new results.

The content of a header box is used only when the quantity is a result of calculation. In this case the calculated values are placed into the indicated column and marked with the indicated header in the first row. If a quantity is an argument then the content of the header box is ignored. Thus the three first header boxes (p, T, and rho) are redundant and never used while the assortment of control buttons is not extended in the future development of the macro.

### Example 1

The starting point is an empty spreadsheet.  $p = 1000$  and  $T = 10$  are considered as constants. Two columns will be filled with generated data and 4 columns with results of calculations.

Six operations are described in a brief protocol:

- 1) Column B, header d:nm, first value 0.31622, last value 10  
Generate geometric progression
- 2) Column C, header rho:g/cm<sup>3</sup>, first value 2.1, last value 1.1  
Generate geometric progression
- 3) Extension 0.3, quantities d : B (header d), Z : D (header Z03)  
Read control data, d ==> Z
- 4) Extension 1995, quantities rho = 1.5 (rho), d : B (d), Z : E (Z95/1.5)  
Read control data, d ==> Z
- 5) Extension 1995, quantities rho = C (rho), d : B (d), Z : F (Z95/var)  
Read control data, d ==> Z
- 6) Extension 0.3, quantities rho = C (rho), d : G (d95v/03), Z : F (Z95/var)  
Read control data, Z ==> d
- 7) EXIT

*Explanation:* the last part of operation 6 "Z : F (Z95:var)" means that in the frame of quantities the values of Z are to be taken from column F of the spreadsheet, where the header is shown as **z95:var**. Z is an argument of operation **z ==> d** and its header is actually not used. The result is d, which header **d95v/03** in column G marks the diameter translated from a modified (variable density) Tammet 1995 model back according to a constant-updated Millikan model with diameter extension of 0.3 nm. The resulting spreadsheet is:

|    | B      | C                     | D       | E       | F       | G       |
|----|--------|-----------------------|---------|---------|---------|---------|
| 1  | d:nm   | rho:g/cm <sup>3</sup> | Z03     | Z95/1.5 | Z95/var | d95w/03 |
| 2  | 0.3162 | 2.1                   | 5.376   | 4.048   | 3.647   | 0.4483  |
| 3  | 0.3981 | 2.011                 | 4.19    | 3.124   | 2.922   | 0.5361  |
| 4  | 0.5012 | 1.927                 | 3.182   | 2.437   | 2.347   | 0.633   |
| 5  | 0.631  | 1.845                 | 2.357   | 1.894   | 1.859   | 0.7485  |
| 6  | 0.7943 | 1.767                 | 1.707   | 1.455   | 1.443   | 0.8903  |
| 7  | 1      | 1.693                 | 1.21    | 1.097   | 1.094   | 1.067   |
| 8  | 1.259  | 1.621                 | 0.8419  | 0.7971  | 0.7962  | 1.303   |
| 9  | 1.585  | 1.553                 | 0.5763  | 0.5154  | 0.5152  | 1.694   |
| 10 | 1.995  | 1.487                 | 0.3892  | 0.3253  | 0.3253  | 2.211   |
| 11 | 2.512  | 1.425                 | 0.2596  | 0.216   | 0.216   | 2.784   |
| 12 | 3.162  | 1.365                 | 0.1715  | 0.146   | 0.146   | 3.454   |
| 13 | 3.981  | 1.307                 | 0.1124  | 0.0983  | 0.09831 | 4.279   |
| 14 | 5.012  | 1.252                 | 0.07319 | 0.06561 | 0.06561 | 5.313   |
| 15 | 6.31   | 1.199                 | 0.04742 | 0.04341 | 0.04341 | 6.611   |
| 16 | 7.943  | 1.148                 | 0.03062 | 0.02852 | 0.02852 | 8.244   |
| 17 | 10     | 1.1                   | 0.01971 | 0.01862 | 0.01862 | 10.3    |

### Example 2

The problem is to study the effect of temperature on mobility of fixed size particles. Change the number of rows to 21 and fill the control boxes of the generator in the following way: A, Celsius, -50, 50. Click "Generate arithmetic progression", mark the diameter as a constant and associate mobility with column B. Click "Read control data" and  $d \Rightarrow Z$ , and use the data in columns A and B for drawing a diagram, which shows dependence of mobility on the temperature. Next, the columns C, D, ... can be filled with temperature diagrams for particles of different sizes, which allows to get several temperature-mobility curves in one diagram.

## References

- Ku, B. K. and Fernandez de la Mora, J. (2009) Relation between Electrical Mobility, Mass, and Size for Nanodrops 1–6.5 nm in Diameter in Air. *Aerosol Sci. Technol.*, **43**, 241–249.
- Li, Z. and Wang, H. (2003) Drag Force, Diffusion Coefficient, and Electric Mobility of Small Particles. II. Application. *Phys. Rev.*, **68**, E061207.
- Mason, E. A. (1984) “Ion Mobility: Its Role in Plasma Chromatography”. In *Plasma Chromatography*, Edited by: Carr, T. W. 43–93. New York and London: Plenum Press.
- Tammet, H. (1995) Size and mobility of nanometer particles, clusters and ions. *J. Aerosol Sci.* **26**, 459–475, <http://www.sciencedirect.com/science/article/pii/002185029400121E>. (Temporarily available <http://1drv.ms/1mABrrE>)
- Tammet, H. (2012a) The function-updated Millikan model: a tool for nanometer particle size-mobility conversions. *Aerosol Sci. Technol.*, **46**, 10, i–iv. Free download <http://dx.doi.org/10.1080/02786826.2012.700740>.
- Tammet, H. (2012b) *Relationship between mobility and diameter of singly charged spherical symmetric aerosol particles*. Workshop presentation slides. <http://1drv.ms/1mAQWLz>.

## Appendix

### User-written algorithm of diameter extension

An advanced user can open the macro *Mob\_size* for editing and enter the code for calculating the diameter extension for a function-updated Millikan model, the principle of which is explained by Tammet (2012a). The raw macro contains a prepared frame of the function:

```
Private Function User_extension() As Double
    Dim y As Double
    User_extension = 0 'temporary replacement
End Function
```

Here the result is zero, which provides the same result as the plain Millikan model. When developing the function the values of global variables *t\_val*, *p\_val* and *rho\_val* can be used. An example is the neighbor function of diameter extension for the model with code 2012:

```
Private Function delta2012() As Double
    Dim y As Double
    y = d_val + 0.00135 * t_val - 1.28
    delta2012 = 0.611 - 0.205 * Exp(-2.4 * y * y) - 0.071 / d_val
End Function
```